

Problem 1: Equilibrium points - Solution

When finding the equilibrium points of this discrete dynamical system, we want to find the fixed points by solving the equation $x_{n+1} = x_n$. This renders $f(x_n)$ equal to zero. One could also note this by recognizing that this system is actually Newton's method for finding roots of a differentiable function. We only consider functions of the form $f(x) = ax^2 + bx = c$, for which we can find roots by just finding the analytical solution

$$x_{1,2}^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (1)$$

The number of distinct roots is determined by the discriminant $b^2 - 4ac$. Therefore, we first calculate the discriminant and then find all the real roots.

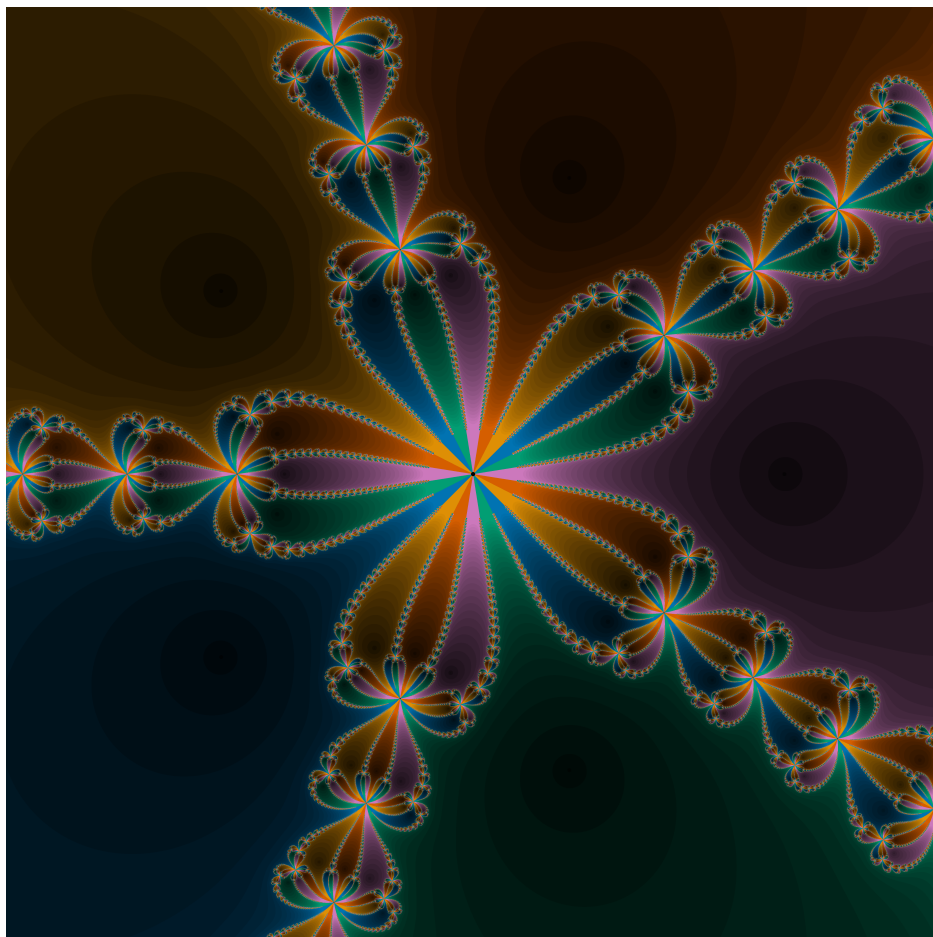


Figure 1: Roots of the polynomial $z^5 - 1 = 0$.